Motion planning problem classification

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1 Problem characteristics

State representation w.r.t. the time derivative:

- (x) - Kinematic (position)
- $(x, \dot{x})$ - Kino-dynamic (position and velocity)
- $(x, \dot{x}, \ddot{x}, F, \tau)$ - Dynamic (position, velocity, acceleration, forces, torques)

Planning space:

- q - Joint space (joint angles)
- y - Task space (base pose, end-effector pose, ...)

Output type:

- x^* - position
- $x^*_{0:T}$ - time indexed trajectory (with different time parametrizations)
- $x^*_{0:n}$ - plain (non-timed) trajectory

Output type w.r.t. the control type:

- x - position
- $\dot{x}$ - velocity
- $\ddot{x}$ - acceleration
- τ - force/torque

Problem type:

- $\underset{x}{\text{argmin}}(x^\top Q x)$ - Unconstrined problem (e.g. Newton step minimization)
- $\underset{x}{\text{argmin}}(Qx + b)$ - Linear programming problem
- $\underset{x}{\text{argmin}}(x^\top Q x + c^\top x), \text{s.t.} \leq b$ - QP problem (with equality an inequality constraints)
- $\underset{x}{\text{argmin}}(f(x)^\top Q f(x) + c^\top f(x), \text{s.t.} Ag(x) \leq b$ - Sequntial QP problem
- $\underset{x}{\text{argmin}} \|f(x)\|^2, \text{s.t.} g(x) \leq 0$ - NLP
- $\underset{x}{\text{argmin}} \|f(x, i)\|^2, \text{s.t.} g(x, i) \leq 0$ - Mixed integer NLP
- $\underset{x}{\text{argmin}} f(x) = \text{True}$ - Sampling problem (uni- or bi- directional)
- $\underset{x}{\text{argmin}} \|f(x)\|^2, \text{s.t.} g(x) = \text{True}$ - Sampling optimization problem

Problem type could also be Hierarchical, e.g. Hierarchical Unconstrained or Hierarchical QP.

Constraint/cost term type:

- $g(x) = c$ - constant
- $g(x) = Ax + b$ - linear/affine
- $g(x) = x^\top Ax$ - squared/quadratic
- $g(x)$ - non-linear

Constraint/cost term differentiability:

- $g(x)$ - non-differentiable (e.g. collision check)
- $\frac{\partial g(x)}{\partial x}$ - 1st order differentiable
- $\frac{\partial^2 g(x)}{\partial^2 x}$ - 2nd order differentiable

2 Naming convention

Problem type	Planning space (if not joint space)	State type (if not Kinematic)	Output type (if not plain trajectory)	Other
[Unconstrained]	[ ]	[ ]	[EndPose]	[ ]
[Unconstrained]	[ ]	[ ]	[TimeIndexed]	[ ]
[Sampling]	[ ]	[ ]	[ ]	[ ]
[SQP]	[ ]	[Dynamic]	[TimeIndexed]	[ ]

The above problems will translate into:

- UnconstrainedEndPoseProblem (IK problem)
- UnconstrainedTimeIndexedProblem (AICO problem)
- SamplingProblem (OMPL problem)
- SQPDynamicTimeIndexedTorqueControlledProblem