qpOASES User's Manual

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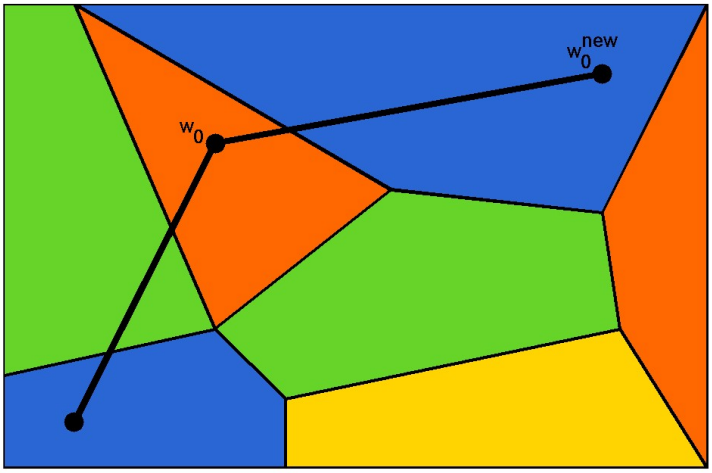

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Chapter 1

Introduction

Model predictive control (MPC) is an advanced control strategy which allows to determine inputs of a given process that optimise the forecasted process behaviour. These inputs, or control actions, are calculated repeatedly using a mathematical *process model* for the prediction. In doing so, the fast and reliable solution of convex *quadratic programming* problems in real-time becomes a crucial ingredient of most algorithms for both linear and nonlinear MPC. The success of linear MPC—where just one *quadratic program (QP)* needs to be solved at each sampling instant—can even be attributed to the fact that highly efficient and reliable methods for QP solution have existed for decades, and that their computation times are much smaller than the required sampling times in typical applications.
On the other hand, quadratic programs also arise as subproblems in sequential quadractic programming (SQP) methods, which require not only one but several QPs be solved during the iteration. SQP methods can be used for solving general nonlinear programs (NLPs) and are also an established tool for solving nonlinear MPC problems.

qpOASES is an open-source implementation of the recently proposed online active set strategy (see [3] and [5]; the main idea has been published earlier for a different class of problems in [2]). It was inspired by important observations from the field of parametric quadratic programming and builds on the expectation that the optimal active set does not change much from one quadratic program to the next. It has several theoretical features that make it particularly suited for model predictive control applications. The software package qpOASES implements these ideas and also incorporates important modifications to make the algorithm numerically more robust [6].
qpOASES solving QPs of the following form:

$$\begin{aligned} \min_{x} \quad & \tfrac{1}{2} x^T H x + x^T g(w_0) \\ \text{s.t.} \quad & \mathrm{lb}A(w_0) \leq Ax \leq \mathrm{ub}A(w_0)\,, \\ & \mathrm{lb}(w_0) \leq \ x \ \leq \mathrm{ub}(w_0)\,, \end{aligned}$$

where the Hessian matrix is symmetric and positive (semi-)definite and the gradient vector as well as the bound and constraint vectors depend affinely on the parameter w_0.

This manual is organised as follows: first, the installation of the qpOASES software package is explained in Chapter 2. Afterwards, a concise description of its main functionality is

given in Chapter 3, which should enable you to use qpOASES within a couple of minutes. Chapter 4 describes special variants for QPs with varying matrices, simply bounded QPs as well as QPs with semi-definite Hessian matrices. Advanced functionality like obtaining status information, using the concept of a so-called initialised homotopy or exploiting QP sparsity is discussed in Chapter 5. Various interfaces to third-party software are presented in Chapter 6. Finally, Chapter 7 (which is mainly intended for software developers) provides some insight into the internal programming structure of qpOASES and options for further tuning of the algorithm.

Further information and a list of frequently asked questions can be found on

http://www.qpOASES.org/ .

If you have got questions, remarks or comments on qpOASES, send them to

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Also bug reports and source code extensions are most welcome!

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Moreover, I owe many thanks to all my former colleagues in Heidelberg and my new ones in Leuven for inspiring discussions and pleasant conversations that were more or less related to the qpOASES software package.

Finally, I would like to thank Peter Ortner, Peter Langthaler and Luigi del Re, Institute for Design and Control of Mechatronical Systems at JKU Linz, for making possible the first run of qpOASES on real (Diesel engine) controller hardware.

The initial version of the software has been partly developed within the framework of the REGINS-PREDIMOT European project whose financial support is acknowledged. Further development of the code has been supported by Research Council KUL: CoE EF/05/006 Optimization in Engineering Center (OPTEC). The main author currently holds a PhD fellowship of the Research Foundation – Flanders (FWO) whose financial support and permission to work on this open-source software project is gratefully acknowledged.

Leuven, 2008
Hans Joachim Ferreau

Chapter 2

Installation

The software package qpOASES is written in an object-oriented manner in C++ and comes along with fully commented source code files. Besides some standards libraries[1] *no further external software packages are required*. Optionally, the LAPACK and BLAS libraries can be linked for performing internal linear algebra operations.

For installing qpOASES under LINUX, perform the following steps:

1. *Download the current version of* qpOASES from

   http://www.qpOASES.org/

   by saving the file `qpOASES-3.0beta.tar.gz` on your local machine.

2. *Unpack the archive:*

   ```
   gunzip qpOASES-3.0beta.tar.gz
   tar xf qpOASES-3.0beta.tar
   ```

   A new directory `qpOASES-3.0beta` will be created; from now on we refer to (the full path of) this directory by <install-dir>. It contains five subfolders, namely

   - `src` (qpOASES source files),
   - `include` (qpOASES header files),
   - `examples` (example files for setting up your own QP problems),
   - `interfaces` (interfaces to third-party software),
   - `doc` (this manual and a DOXYGEN configuration file).

3. qpOASES is distributed under the terms of the GNU Lesser General Public License 2.1, which you can find in the file <install-dir>/LICENSE.txt or Appendix A of this manual. *Please read this licence file carefully before you proceed with the installation*, as you implicitly agree with this licence by using qpOASES!

[1] `math.h`, `stdio.h`, `string.h` (as well as `sys/time.h`, `sys/stat.h` or `windows.h` for runtime measurements)

4. If you want to use qpOASES via the provided third-party interfaces only, you can skip the following steps and proceed as described in Chapter 6. Otherwise continue with the

   Compilation of the qpOASES *library* libqpOASES.a[2]:

   ```
   cd <install-dir>/src
   make
   ```

   This library libqpOASES.a provides the complete core functionality of the qpOASES software package. It can be used by, e.g., linking it against a main function from the examples folder.

5. *Compilation of a set of simple test examples:*

   ```
   cd <install-dir>/examples
   make
   ```

   Among others, an executable called example1 should have been created; run it in order to test your installation. If it terminates after successfully solving two QPs, qpOASES has been successfully installed!

6. *Optional, create source code documentation*[3]*:*

   ```
   cd <install-dir>/doc
   doxygen doxygen.config
   ```

   Afterwards, you can open the file <install-dir>/doc/html/index.html with your favorite browser in order to view qpOASES's source code documentation.

Remarks:

- It is also possible to install qpOASES on a WINDOWS or MAC OS machine as it does not require LINUX-specific commands.
- If compilation fails due to the fact that the snprintf() function is not supported, you might uncomment line 41 within <install-dir>/include/Types.hpp and try to compile again.

[2] The make command also creates a library called libqpOASESextras.a whose meaning is described in Section 5.8.

[3] All source code files are commented in a way suitable for the documentation system DOXYGEN [7].

Chapter 3

Getting Started

This chapter explains to you within a few minutes how to solve a quadratic programming (QP) problem, or a whole sequence of them, by means of qpOASES. At the end a tutorial example is presented that might serve as a template for your own QPs.

3.1 Outline

Core of qpOASES is the QProblem class which is able to store, process and solve convex quadratic programs using the online active set strategy; it makes use of several auxiliary classes (see Chapter 7). Except for special situations, the QProblem class is intended to be the only *user interface* to qpOASES's functionality.

For solving a series of convex quadratic programs with fixed Hessian and constraint matrix, the following steps are necessary:

1. create an instance of the QProblem class,
2. initialise your QProblem object and solve the first QP (specified by its QP matrices and vectors),
3. solve each following QP by passing its vectors to your QProblem object.

Now, we will explain these three steps in more detail. Various variants and special cases are treated in later chapters for the ease of presentation.

3.2 Main Steps

Creating an Instance of the QProblem Class

Creating an QProblem object is done by means of the following constructor

```
QProblem( int nV, int nC );
```

which takes the number of variables nV and the number of constraints nC of the quadratic program sequence to be solved. At the moment it is not possible to solve QP sequences with varying problem dimensions.

Summary of the first step:
You can create an instance `example` of the `QProblem` class with the following command:
`QProblem example( nV,nC );`

Initialisation and Solution of First QP

The second step requires to initialise all internal data structures of the `QProblem` object and the solution of the first QP. Both can be accomodated with a single call to the following function:

```
returnValue init( const double* const H,
                  const double* const g,
                  const double* const A,
                  const double* const lb,
                  const double* const ub,
                  const double* const lbA,
                  const double* const ubA,
                  int&                nWSR,
                  double* const       cputime
                  );
```

which takes the positive (semi-)definite Hessian matrix $H \in \mathbb{R}^{\mathrm{nV} \times \mathrm{nV}}$, the gradient vector $g \in \mathbb{R}^{\mathrm{nV}}$, the constraint matrix $A \in \mathbb{R}^{\mathrm{nC} \times \mathrm{nV}}$ the lower and upper bound vectors $\mathrm{lb}, \mathrm{ub} \in \mathbb{R}^{\mathrm{nV}}$ and the lower and upper constraints' bound vectors $\mathrm{lb}A, \mathrm{ub}A \in \mathbb{R}^{\mathrm{nC}}$ of the initial quadratic program. Equality constraints are imposed be setting the corresponding entries of lower and upper (constraints') bounds vectors to the same value.
All these data must be stored in arrays of type `double` (matrices stored row-wise, i.e. C style, in an one-dimensional array) with appropriate dimensions. If there are, for example, no upper bounds in your QP formulation, you can pass a null pointer instead of vector lb[1].
All `init` functions make deep copies of all arguments, thus afterwards you have to free their memory yourself.
The function `init` initialises all internal data structures, e.g. matrix factorisations, and *solves* the first quadratic program using the initial homotopy idea of the online active set strategy. The integer argument `nWSR` specifies the maximum number of working set recalculations to be performed during the initial homotopy (on output it contains the number of working set recalculations actually performed!). If `cputime` is not the null pointer, it contains the *maximum allowed* CPU time in seconds for the whole initialisation (and the actually required one on output). See Section 5.6 for further details.

The function `init` returns a status code (of type `returnValue`) which indicates whether the initialisation was successful; possible values are:

- `SUCCESSFUL_RETURN`: initialisation successful (including solution of first QP),
- `RET_MAX_NWSR_REACHED`: initial QP could not be solved within the given number of working set recalculations,

[1] If your QP does not comprise constraints (apart from bounds), you should make use of a special variant for simply bounded QPs (cf. Chapter 4).

- RET_INIT_FAILED (or a more detailed error code): initialisation failed.

If init indicates a SUCCESSFUL_RETURN, several functions enable you to obtain information about the solution of the first QP. The most important ones are:

- `returnValue getPrimalSolution( double* const xOpt ) const`

  that writes the optimal primal solution vector (dimension: nV) into the array xOpt, which has to be allocated (and freed) by the user;

- `returnValue getDualSolution( double* const yOpt ) const`

  that writes the optimal dual solution vector[2] (dimension: $\mathrm{nV} + \mathrm{nC}$) into the array yOpt, which has to be allocated (and freed) by the user;

- `double getObjVal( ) const`

  that returns the optimal objective function value.

Summary of the second step:
Having created an QProblem object example, it can be initialised together with solving the first QP with the following command: `example.init( H,g,A,lb,ub,lbA,ubA,nWSR,cputime );`

Solution of the Following QPs

If not only a single quadratic program but a whole sequence of QPs shall be solved—as it is the usual situation for an MPC problem—the next QP can be solved using the function:

```
returnValue hotstart( const double* const g_new,
                      const double* const lb_new,
                      const double* const ub_new,
                      const double* const lbA_new,
                      const double* const ubA_new,
                      int&                nWSR,
                      double* const       cputime
                      );
```

The next QP is specified by passing its gradient vector g_new, its lower and upper bound vectors lb_new and ub_new as well as its lower and upper constraints' bound vectors lbA_new and ubA_new (QP matrices are assumed to be constant). It is solved by means of the online active set strategy using at most nWSR working set recalculations or at most cputime seconds of CPU time (if not null). On output nWSR and cputime contain the number of

[2] We use the following definition of the Lagrange function to define the dual multipliers:

$$Hx^{\mathrm{opt}} + g(w_0) - A^T y^{\mathrm{opt}} = 0 \quad \Longleftrightarrow \quad \mathtt{H \cdot x + g - y[0...nV\text{-}1] - A^T y[nV...nV+nC\text{-}1]} = 0$$

The dual solution vector contains exactly one entry per lower/upper bound as well as exactly one entry per lower/upper constraints' bound. Positive entries correspond to active lower (constraints') bounds, negative entries to active upper (constraints') bounds and a zero entry means that both corresponding (constraints') bounds are inactive.

working set recalculations that were actually performed and the actually required CPU time for solving the next QP, respectively.

Like most qpOASES functions, hotstart returns a status code; possible values are:

- SUCCESSFUL_RETURN: QP has been solved,
- RET_MAX_NWSR_REACHED: QP could not be solved within the given number of working set recalculations,
- RET_HOTSTART_FAILED (or a more detailed error code): QP solution failed.

Summary of the third step:
Having created and initialised a QProblem object example, the next QP can be solved as follows: `example.hotstart( g_new,lb_new,ub_new,lbA_new,ubA_new,nWSR,cputime );`

3.3 A Tutorial Example

A complete example for solving two very simple quadratic programs using qpOASES is given in the file <install-dir>/examples/example1.cpp:

```
#include <qpOASES.hpp>

int main( )
{
    USING_NAMESPACE_QPOASES

    /* Setup data of first QP. */
    double H[2*2] = { 1.0, 0.0, 0.0, 0.5 };
    double A[1*2] = { 1.0, 1.0 };
    double g[2] = { 1.5, 1.0 };
    double lb[2] = { 0.5, -2.0 };
    double ub[2] = { 5.0, 2.0 };
    double lbA[1] = { -1.0 };
    double ubA[1] = { 2.0 };

    /* Setup data of second QP. */
    double g_new[2] = { 1.0, 1.5 };
    double lb_new[2] = { 0.0, -1.0 };
    double ub_new[2] = { 5.0, -0.5 };
    double lbA_new[1] = { -2.0 };
    double ubA_new[1] = { 1.0 };

    /* Setting up QProblem object. */
    QProblem example( 2,1 );

    /* Solve first QP. */
    int nWSR = 10;
    example.init( H,g,A,lb,ub,lbA,ubA, nWSR,0 );

    /* Solve second QP. */
```

```
    nWSR = 10;
    example.hotstart( g_new,lb_new,ub_new,lbA_new,ubA_new, nWSR,0 );

    /* Get and print solution of second QP. */
    double xOpt[2];
    example.getPrimalSolution( xOpt );
    printf( "\n xOpt = [ %e, %e ];  objVal = %e\n\n",
                          xOpt[0],xOpt[1],example.getObjVal() );

    return 0;
}
```

In order to access the functionality of the qpOASES software package via the `QProblem` class, the header file `QProblem.hpp` is included.
The `main` function starts with defining the data of two very small-scale QPs. Afterwards, a `QProblem` object is created which is then initialised together with solving the first QP. Finally, the `hotstart` function is used to solve the second QP.
You might wonder about the command `using namespace qpOASES;` at the very top of the `main` function. It is used because all classes, global functions and variables of the qpOASES software package are *collected in a common namespace* that is called qpOASES, too.

3.4 Setting Up Your Own Example

The easiest way for setting up your own example, say `yourexample`, is to use an existing one as a template. In doing so, perform the following steps:

1. *Copy the existing example:*

   ```
   cd <install-dir>/examples
   cp example1.cpp yourexample.cpp
   ```

2. *Edit the examples* `Makefile`:

   Open the file `<install-dir>/examples/Makefile` and add a new target

   ```
   yourexample:  yourexample.o
       ${CPP} -o $@ ${CPPFLAGS} $@.o -L${LIBS_PATH} -l${QPOASES_LIB}
   ```

   (Do not forget to add its name to the `all` target.)

3. *Implement your own example:*

   Modify your file `<install-dir>/examples/yourexample.cpp` and run `make`. An executable called `yourexample` should be at your service.

Chapter 4

Solution Variants for Special QP Types

qpOASES is a structure-exploiting active-set QP solver. This chapter details how to most efficiently solve your QPs with qpOASES by choosing a solution variant that matches best your specific problem type. For this purpose, three different QProblem-like classes with overloaded constructors are available.

4.1 Solving QPs in Standard Form

Usually qpOASES expects QPs to be formulated in the following *standard form*:

$$\begin{aligned} \min_{x} \quad & \tfrac{1}{2} x^T H x + x^T g(w_0) \\ \text{s.t.} \quad & \mathrm{lb}A(w_0) \leq Ax \leq \mathrm{ub}A(w_0)\,, \\ & \mathrm{lb}(w_0) \leq \ x \ \leq \mathrm{ub}(w_0)\,, \end{aligned}$$

with a positive (semi-)definite Hessian matrix H. If your QP is given in exactly this form, you should simply make use of the standard QProblem class as described in Chapter 3. Otherwise, solving your QP is also possible or can be done more efficiently if

- also your QP matrices H and/or A are varying from one QP to the next by using the SQProblem class (see Section 4.2);
- your QP formulation does *not* comprise constraints involving a matrix A by using the QProblemB class (see Section 4.3);
- your Hessian matrix H is not positive definite but only positive semi-definite by using a dedicated constructor (see Section 4.4);
- your Hessian matrix H is zero, i.e. your QP is actually a linear program, by using a dedicated constructor (see Section 4.5);
- your Hessian matrix H happens to be the identity matrix by using a dedicated constructor (see also Section 4.5).

4.2 Solving QPs with Varying Matrices

Although the online active set strategy was originally designed for QP sequences with fixed Hessian and constraint matrices, it can be extended to the case where also these matrices vary from QP to the next (see [4] for a mathematical description of this idea).
In order to use this extension, two modifications are necessary:

1. Create an instance of the `SQProblem` class (instead of one of type `QProblem`) by using the constructor of the `SQProblem` class and a suitable `init` function. Both take *exactly the same* arguments as those of the `QProblem` class.

2. Call the modified function

   ```
   returnValue hotstart( const double* const H_new,
                         const double* const g_new,
                         const double* const A_new,
                         const double* const lb_new,
                         const double* const ub_new,
                         const double* const lbA_new,
                         const double* const ubA_new,
                         int&                nWSR,
                         double* const       cputime
                         );
   ```

   for transition from one QP to the next; it also takes the new Hessian `H_new` as well as the new constraint matrix `A_new` as arguments.

A complete example for using the `SQProblem` class can be found within the file `<install-dir>/examples/example1a.cpp`.

4.3 Solving Simply Bounded QPs

We call a quadratic program "simply bounded" whenever it does not comprise constraints but only bounds:

$$\begin{aligned} \min_{x} \quad & \tfrac{1}{2} x^T H x + x^T g(w_0) \\ \text{s.t.} \quad & \mathrm{lb}(w_0) \leq \ x \ \leq \mathrm{ub}(w_0) \,. \end{aligned}$$

This special form can be exploited within the solution algorithm for speeding up the computation, typically by a factor of three to five. Therefore, the qpOASES software package implements the special class `QProblemB` for solving simply bounded QPs.

In order to make use of this feature do the following:

1. You have to create a `QProblemB` object using the following constructor

   ```
   QProblemB( int nV );
   ```

2. Afterwards you can initialise the `QProblemB` object together with solving the first simply bounded QP by calling, for example,

```
returnValue init( const double* const H,
                  const double* const g,
                  const double* const lb,
                  const double* const ub,
                  int&                nWSR,
                  double* const       cputime
                  );
```

The only difference from the QProblem class is the fact that the arguments specifying the constraints—i.e. A, lbA, ubA, and nC—are missing.

3. For solving the next problem within your QP sequence, the following variant of the hotstart function is available:

```
returnValue hotstart( const double* const g_new,
                      const double* const lb_new,
                      const double* const ub_new,
                      int&                nWSR,
                      double* const       cputime
                      );
```

Again, it takes exactly the same arguments as the corresponding QProblem member function except for the two arguments lbA_new, ubA_new.

A complete example for using the QProblemB class can be found within the file <install-dir>/examples/example1b.cpp.

4.4 Solving QPs with Positive Semi-Definite Hessian Matrix

qpOASES provides two different strategies to deal with semi-definite QPs. All mentioned options to enable and to adjust these strategies are described in more detail in Section 5.2.

Automatic Regularisation Procedure

The first one is a regularisation procedure that is computationally cheap and works well for many problems. This procedure first adds a small multiple of the identity matrix[1] to the Hessian and solves the corresponding regularised QP. Afterwards, a few post-iterations[2] are performed that improve solution accuracy signigificantly over a plain regularisation at virtually now extra computational cost. If your QP involves a Hessian matrix that is only positive semi-definite, this regularisation scheme *is used automatically*, i.e. without any change in the constructor or other function calls, whenever the option enableRegularisation is set to BT_TRUE.

Although semi-definiteness can be easily detected, this causes a certain computational overhead[3] that can be avoided by a dedicated constructor call, e.g.:

[1] given by the option epsRegularisation
[2] given by the option numRegularisationSteps
[3] an additional Cholesky decomposition

```
QProblem( int nV, int nC, HessianType hessianType );
```

Therein, hessianType can take one of the following values:

- HST_POSDEF: Hessian matrix is positive definite,
- HST_SEMIDEF: Hessian matrix is positive semi-definite,
- HST_ZERO: Hessian matrix is zero matrix (see next section),
- HST_IDENTITY: Hessian matrix is identity matrix (see next section).

If hessianType is set to HST_SEMIDEF or HST_ZERO, the built-in regularisation scheme is switched on at no additional computational cost. Corresponding overloaded constructors also exist for the SQProblem and QProblemB class, respectively.

Nonzero Curvature Tests and Flipping Bounds

A second strategy to deal with semi-definite QPs is the use of nonzero curvature tests as described in [2]. The main idea is to check upon removal of an active constraint or bound whether the projected Hessian matrix will loose full rank. If so, another constraint or bound is immediately added to the active set to ensure full rank rank of the projected Hessian matrix. Nonzero curvature tests can be enabled by setting the option enableNZCTests to BT_TRUE.

Nonzero curvature tests can be combined with the use of flipping bounds as proposed in [6]: In case removal of an active constraint or bound causes the smallest eigenvalue of the projected Hessian matrix to drop below a small positive threshold, the constraint or bound remains active but the intermediate QP data is changed such that it is active at its opposite limit (e.g. an active upper bound will become an active lower bound). This also prevents the Cholesky decomposition from becoming ill-conditioned in case the Hessian matrix is positive definite with very small positive eigenvalues. Flipping bounds can be enabled by setting the option enableFlippingBounds to BT_TRUE. An option epsFlipping can be used to adjust the lower threshold allowed for the smallest positive eigenvalue of the projected Hessian matrix.

The flipping bound strategy requires the initial projected Hessian matrix to be positive definite. The option initialStatusBounds provides an easy way to ensure this by initially fixing all bound constraints to their respective lower or upper limit (that way the projected Hessian matrix has zero dimension). Alternatively, an initial guess for the active set as described in Section 5.5 might be used to ensure positive definiteness of the projected Hessian matrix.

4.5 Solving QPs with Trivial Hessian Matrix

Whenever a Hessian matrix is passed to qpOASES, i.e. when calling a init function or performing a hotstart while using the SQProblem class, it is internally checked whether the Hessian is trivial. It is considered trivial if and only if it is the zero or identity matrix,

corresponding to HST_ZERO or HST_IDENTITY as mentioned in the previous section. If the Hessian is trivial, several simplifications of the internal linear algebra operations apply, cutting computational load by about a factor of two.

If your Hessian is trivial, you might explicitly provide this information to qpOASES via a dedicated constructor call, e.g.,

```
QProblem( int nV, int nC, HessianType hessianType );
```

(corresponding overloaded constructors also exist for the SQProblem and QProblemB class, respectively). If you set hessianType to HST_ZERO or HST_IDENTITY, no internal memory for storing the Hessian matrix is allocated. Moreover, when doing so you are allowed to pass a null pointer as argument within all function calls involving the Hessian matrix, e.g.,

```
// assumes that a QProblem object "qp" exists
qp.init( 0,g,A,lb,ub,lbA,ubA,nWSR,cputime );
```

A null pointer is then interpreted as zero or identity matrix, respectively. Whenever you pass a non-null argument, a full Hessian matrix is expected and its type is automatically determined internally.

Solving Linear Programming (LP) Problems

Both strategies mentioned in Section 4.4 in principle also allow to solve linear programming (LP) problems by means of qpOASES. However, qpOASES *is not a dedicated (parametric) LP solver*, thus using it for solving LPs might be (highly) inefficient due to the dense linear algebra or might even fail in certain circumstances. Therefore, this additional feature should be only used for small-scale LPs (comprising not more than, say, hundreds of variables) and in situations where computational time is not the main concern.

A complete example for solving two small-scale LPs with qpOASES can be found within the file <install-dir>/examples/exampleLP.cpp.

Solving QPs whose Hessian is the Identity Matrix

Via a coordinate transformation, every strictly convex QP can be transformed into an equivalent one whose Hessian is the identity matrix. Also ℓ_2-norm minimisation problems naturally pose QPs whose Hessian is the identity matrix. Thus, it is possible to provide such a QP sequence to qpOASES by specifying the Hessian type to be HST_IDENTITY within the above-mentioned constructor call; all other function calls remain unaltered.

Chapter 5

Advanced Functionality

5.1 Obtaining Status Information

There are many functions for obtaining status information on the current iterate. Firstly, you can obtain the primal and dual iterate as well as the corresponding objective function value by using, respectively:

- `returnValue getPrimalSolution( double* const xOpt ) const`,
- `returnValue getDualSolution( double* const yOpt ) const`,
- `double getObjVal( ) const`.

If you wonder why these are the same functions as for obtaining the optimal solution after a QP has been solved (cf. Section 3.2), you should recall that qpOASES uses a homotopy for solving the current QP that produces a sequence of iterates that are *optimal for intermediate QPs* along the homotopy path.
The first two functions expect an allocated `double` array and store the optimal solution vector if and only if the (intermediate) QP has been solved; otherwise the error code `RET_QP_NOT_SOLVED` is returned. The function `getObjVal( )` calculates and returns the optimal objective function value or returns `INFTY` if the (intermediate) QP has not been solved.

Secondly, you can ask for the total number of variables and constraints and for the cardinality of certain subsets (at current iterate!) of them:

- `int getNV( ) const`: returns number of variables,
- `int getNFR( ) const`: returns number of free variables,
- `int getNFX( ) const`: returns number of fixed variables,
- `int getNC( ) const`: returns number of constraints,
- `int getNEC( ) const`: returns number of (implicitly defined) equality constraints,
- `int getNAC( ) const`: returns number of active constraints,

- `int getNIAC( ) const` : returns number of inactive constraints.

  Moreover,

- `int getNZ( ) const` : returns dimension of the null space of active constraints.

Finally, you can ask for the overall status of the QP (object):

- `BooleanType isInitialised( ) const` : returns `BT_TRUE` if and only if the QP object has been initialised,
- `BooleanType isSolved( ) const` : returns `BT_TRUE` if and only if QP has been solved,
- `BooleanType isInfeasible( ) const` : returns `BT_TRUE` if and only if QP was found to be infeasible.

5.2 Options for Solving QPs

The way qpOASES solves QPs can be adjusted in several ways by means of the class `Options`. It comprises the following members whose values can be set by the user:

Name:	*Possible values:*	*Description:*

`printLevel`	`PL_NONE`
	`PL_LOW`
	`PL_MEDIUM`
	`PL_HIGH`
		Defines the amount of text output during QP solution, see Section 5.7.

`enableRamping`	`BT_TRUE` `BT_FALSE`	Enables or disables ramping, an idea to avoid ties when determining the step length [6].
`enableFarBounds`	`BT_TRUE` `BT_FALSE`	Enables or disables the use of far bounds, an idea to reliably detect unboundedness [6].
`enableFlippingBounds`	`BT_TRUE` `BT_FALSE`	Enables or disables the use of flipping bounds as described in Section 4.4.
`enableRegularisation`	`BT_TRUE` `BT_FALSE`	Enables or disables the Hessian regularisation scheme as described in Section 4.4.
`enableFullLITests`	`BT_TRUE` `BT_FALSE`	Enables or disables a condition-hardened, but more expensive test for linear independence.
`enableNZCTests`	`BT_TRUE` `BT_FALSE`	Enables or disables nonzero curvature tests as described in Section 4.4.
`enableDriftCorrection`	`int` (≥ 0)	Specifies the frequency of drift corrections [6]: 0 turns them off, 1 uses them at each iteration etc.

enableCholeskyRe-factorisation	int (≥ 0)	Specifies the frequency of full refactorisations of the projected Hessian: 0 turns them off, 1 uses them at each iteration etc.
enableEqualities	BT_TRUE BT_FALSE	Specifies whether equalities shall be always treated as active constraints.
terminationTolerance	double (> 0)	Relative termination tolerance to stop homotopy.
boundTolerance	double (> 0)	If upper and lower limits differ less than this tolerance, they are regarded equal, i.e. as equality constraint.
boundRelaxation	double (> 0)	Initial relaxation of bounds to start homotopy and initial value for far bounds.
epsNum	double	Numerator tolerance for ratio tests.
epsDen	double	Denominator tolerance for ratio tests.
maxPrimalJump	double (> 0)	Maximum jump in primal variables allowed in nonzero curvature tests.
maxDualJump	double (> 0)	Maximum jump in dual variables allowed in linear independence tests.
initialRamping	double (> 0)	Start value for ramping strategy.
finalRamping	double (> 0)	Final value for ramping strategy.
initialFarBounds	double (> 0)	Initial size of far bounds.
growFarBounds	double (> 1)	Factor to grow far bounds.
initialStatusBounds	ST_INACTIVE ST_LOWER ST_UPPER	Initial status of bounds at first iteration: all inactive or all active at their lower or upper limits, respectively.
epsFlipping	double (> 0)	Tolerance of squared entry on Cholesky diagonal which triggers flipping bound.
numRegularisationSteps	int (≥ 0)	Maximum number of successive regularisation steps.
epsRegularisation	double (> 0)	Scaling factor of identity matrix used for Hessian regularisation.
numRefinementSteps	int (≥ 0)	Maximum number of iterative refinement steps.
epsIterRef	double (> 0)	Early termination tolerance for iterative refinement [6].
epsLITests	double (> 0)	Tolerance for linear independence tests.
epsNZCTests	double (> 0)	Tolerance for nonzero curvature tests.

If the user does not specify any options, default values are used. For changing these default values, the following steps are required:

1. Create an `Options` object and modify any of the above mentioned options as follows

```
Options myOptions;
myOptions.<optionName> = <optionValue>;
```

2. Pass your options to the QP object:

```
// assumes that a QP object "qp" exists
qp.setOptions( myOptions );
```

In order to facilitate the choice of reasonable values for all these options, the `Options` class offers a couple of pre-defined configurations:

- `setToDefault( );` assigns default values to all options,
- `setToReliable( );` chooses values that ensure maximum reliabilty of the QP solution (usually at the expense of a slower execution),
- `setToFast( );` chooses values that ensure maximum computational speed that might lead to a failure of the algorithm in certain cases.

Thus, a complete example could look like:

```
Options myOptions;
myOptions.setToFast( );
myOptions.printLevel = PL_LOW;
qp.setOptions( myOptions );
```

Note that changing options will take effect immediately after passing them.

5.3 Exploiting Sparsity in Hessian and Constraints Matrix

qpOASES has been developed for small- to medium scale QPs resulting from MPC formulations after the differential states have been eliminated. These QPs usually feature a fully dense Hessian matrix and a lower triangular constraint matrix. Consequently the whole internal linear algebra—including the matrix factorisations—is implemented dense. For enhancing qpOASES's applicability to general QPs, a miminalistic `Matrix` base class has been introduced. This framework also supports sparse QP matrices and allows one to use special linear algebra routines for symmetric matrices.

For passing sparse QP matrices, overloaded variants of all `init` and `hotstart` routines exists. These variants do not read the Hessian and constraints matrix from double arrays but rather expect them in form of derived classes of the minimalistic `Matrix` base class, for example:

```
returnValue init( SymmetricMatrix*    H,
                  const double* const g,
                  Matrix*             A,
                  const double* const lb,
                  const double* const ub,
                  const double* const lbA,
                  const double* const ubA,
                  int&                nWSR,
                  double* const       cputime
                  );
```

General dense matrices are stored within instances of the class `DenseMatrix`, general sparse matrices within ones of the class `SparseMatrix`. For symmetric matrices the classes `SymDenseMat` and `SymSparseMat` are provided, respectively. Sparse matrices are stored in column compressed storage format. We refer to the DOXYGEN source code documentation for further details.

A complete tutorial example illustrating the use of sparse QP matrices can be found within the file `<install-dir>/examples/qrecipe.cpp`.

5.4 Speeding-Up Solution for QPs Comprising Many Constraints

Heuristic for Approximating the Constraint Product

In case the QP comprises much more (dense) constraints than optimisation variables, the step length determination requires a major part of the overall computational load per QP iteration. That is because the (costly) matrix-vector product Ax has to be formed for determining if an inactive constraint is going to become active at the next iterate.
qpOASES has implemented a strategy that only *approximates* this matrix-vector product when inactive constraints are so far off their limits that they cannot become active during the next step. This strategy still ensures exact QP solution and can lead to considerable computational savings. However, in worst-case it can even prolong computation time, thus it needs to be explicitly enabled by defining the compiler flag `__MANY_CONSTRAINTS__`. Note, that this strategy relies on the fact that *each constraint has ℓ_1-norm not greater than 1*! Thus, before setting this compiler flag, you might need to re-scale your constraints (otherwise QP solution can fail!).

Specifying a Function for Evaluating the Constraints

Another possibility to speed-up QP solution in case of many constraints is available whenever the calculation of the matrix-product of the constraint matrix `A` with the current primal iterate `x` can be simplified. In that case, the user can provide a dedicated function that can evaluate the product of any constraint at a given primal iterate. Once such a function is specified and passed to an QP object, qpOASES will use this user-provided function for calculating the constraint products instead of doing a standard (but possibly naive) matrix-vector multiplication.

For using this functionality, you have to perform the following steps:

1. Derive a customized class from the abstract base class ConstraintProduct as declared within <install-dir>/include/ConstraintProduct.hpp. Within this class, you have to implement the function operator which has the following form:

   ```
   virtual int operator()( int constrIndex,
                           const double* const x,
                           double* const constrValue
                           ) const;
   ```

   It takes the index of the constraint to be evaluated (between 0 and nC) and an array containing the current primal iterate (of size nV) as input arguments and writes the corresponding product into constrValue. The function operator needs to return 0 on success and might return an error code otherwise.

2. Make this derived class available within your example, instantiate an object of this class and pass it to the QP object by calling

   ```
   // assumes that a QP object "qp" exists
   MyConstraintProduct myCP( );
   qp.setConstraintProduct( &myCP );
   ```

A full tutorial example illustrating this feature of qpOASES can be found within the file <install-dir>/examples/example4.cpp.

5.5 Initialised Homotopy

For solving a QP, qpOASES always starts at the optimal solution of the previous QP and performs a homotopy to the optimal solution of the QP to be solved. At the very beginning of a sequence (when init is called) an auxiliary QP is constructed internally whose optimal solution is known. This optimal solution serves as a starting point for the homotopy to the optimal solution of the (actual) initial QP. By default, this auxiliary QP has the origin as solution and its active set is empty (or comprising implicitly fixed variables and equality constraints only).

The notion *initialised homotopy* refers to the possibility to incorporate an initial guess for the optimal solution or the active set at the solution into the construction of the auxiliary QP. This is done by calling a special variant of the init function:

```
returnValue init( const double* const      H,
                  const double* const      g,
                  const double* const      A,
                  const double* const      lb,
                  const double* const      ub,
                  const double* const      lbA,
                  const double* const      ubA,
                  int&                     nWSR,
                  double* const            cputime,
```

```
                const double* const      xOpt,
                const double* const      yOpt,
                const Bounds* const      guessedBounds,
                const Constraints* const guessedConstraints
                );
```

Besides the arguments of the usual `init` function, it (optionally) takes guesses for the primal solution vector `xOpt`, the dual solution vector `yOpt` or the status (active/inactive) of bounds and constraints at the solution (see below). Null pointers can be passed for all of these arguments. The construction of the auxiliary QP now depends on the arguments passed (for convenience we summarise `guessedBounds` and `guessedConstraints` to `guess` which is null if and only if both parts are null) as follows:

1. `xOpt == 0, yOpt == 0, guess == 0`: start at primal/dual origin with empty active set (usual auxiliary QP setup);

2. `xOpt != 0, yOpt == 0, guess == 0`: start at primal/dual origin and determine active set by "clipping"[1];

3. `xOpt == 0, yOpt != 0, guess == 0`: start with primal variables equal to zero, dual variables equal to given vector and determine active set from signs of dual variables;

4. `xOpt == 0, yOpt == 0, guess != 0`: start at primal/dual origin and with given active set;

5. `xOpt != 0, yOpt != 0, guess == 0`: start with given vectors for primal and dual variables and determine active set from signs of dual variables;

6. `xOpt != 0, yOpt == 0, guess != 0`: start with primal variables equal to given vector, dual variables equal to zero and with given active set;

7. `xOpt != 0, yOpt != 0, guess != 0`: start with given vectors for primal and dual variables and with given active set (assume them to be consistent!).

The remaining eighth combination is not allowed for consistency reasons.

Besides initialising the homotopy at startup of the QP sequence, it is also possible to incorporate an initial guess for the active set when calling the `hotstart` function:

```
returnValue hotstart( const double* const      g_new,
                      const double* const      lb_new,
                      const double* const      ub_new,
                      const double* const      lbA_new,
                      const double* const      ubA_new,
                      int&                     nWSR,
                      double* const            cputime,
                      const Bounds* const      guessedBounds,
                      const Constraints* const guessedConstraints
                      );
```

[1] i.e. add all bounds and constraints to active set that are violated for given primal solution vector

In this case only the active set can be specified, primal and dual solution vectors are always taken from the previous QP solution. This `hotstart` variant updates the active set according to the user's guess and performs a usual homotopy afterwards.

Specifying an Initial Guess for the Active Set

For specifying an initial guess for the active set, you have to setup a `Bounds` and/or `Constraints` object. This can either be done from scratch or by modifying an exisiting one. For the first variant you might use the following code fragment:

```
// assumes that a QP object "qp" exists
int nV = qp.getNV( );
int nC = qp.getNC( );

Bounds guessedBounds( nV );
guessedBounds.setupAllLower( );

Constraints guessedConstraints( nC );
guessedConstraints.setupAllInactive( );
```

First, a `Bounds` object comprising a working set of `nV` bounds is constructed and afterwards all bounds are set to be active at their lower limit. Second, a `Constraints` object is constructed analogously and all constraints are set to be inactive. For a `Bounds` object you can call one of the following functions:

- `returnValue setupAllFree( )`: all variables are free, i.e. bounds are inactive,
- `returnValue setupAllLower( )`: all variables are fixed at their lower limits,
- `returnValue setupAllUpper( )`: all variables are fixed at their upper limits.

For a `Constraints` object you can call one of the following functions:

- `returnValue setupAllInactive( )`: all constraints are inactive,
- `returnValue setupAllLower( )`: all constraints are active at their lower limits,
- `returnValue setupAllUpper( )`: all constraints are active at their upper limits.

Moreover, you might setup the status of each bound/constraint one by one by calling:

- `returnValue setupBound( int number, SubjectToStatus status )` or
- `returnValue setupConstraint( int number, SubjectToStatus status )`,

repectively, where `number` specifies the number of the repective bound/constraint (starting at zero!) and `status` is one of the following types:

- `ST_INACTIVE`: bound/constraint is inactive,
- `ST_LOWER`: bound/constraint is active at its lower limit,

- ST_UPPER: bound/constraint is active at its upper limit.

Please note that you can call *either* exactly one setupAll* variant *or* exactly one of setupBound/setupConstraint for each single bound/constraint!

Instead of setting up a Bounds/Constraints object from scratch, you might want to *modify an existing one*. For achieving this, you will most commonly first obtain a copy of the active set of the current QP by calling:

```
// assumes that a QP object "qp" exists
Bounds guessedBounds;
qp.getBounds( guessedBounds );

Constraints guessedConstraints;
qp.getConstraints( guessedConstraints );
```

Afterwards you might use one of the following functions to manipulate a Bounds object:

- returnValue moveFixedToFree(int number): moves the number-th bound from the working set of fixed variables to that of free ones,
- returnValue moveFreeToFixed(int number, SubjectToStatus status): moves the number-th bound from the working set of free variables to that of fixed ones (where status must be either ST_LOWER or ST_UPPER).

For a Constraints object you can call one of the following functions:

- returnValue moveActiveToInactive(int number): moves the number-th constraint from the working set of active constraints to that of inactive ones,
- returnValue moveInactiveToActive(int number, SubjectToStatus status): moves the number-th constraint from the working set of inactive constraints to that of active ones (where status must be either ST_LOWER or ST_UPPER).

Moreover, in the model predictive control context it is very common that the active set is *shifted* between two consecutive sampling instants. Therefore, for both Bounds and Constraints you can also call one of the following functions:

- returnValue shift(int offset): shifts forward the working set of bounds/constraints by a given offset (which has to be an integer divisor of the total number of bounds/constraints), i.e. the status information of the first offset bounds/constraints is thrown away and the one of the last offset ones is duplicated;
- returnValue rotate(int offset): rotates forward the working set of bounds/constraints by a given offset.

We refer to the DOXYGEN documentation (cf. installation step six described in Chapter 2) for more details.

5.6 Specifying a CPU Time Limit for QP Solution

For all `init` and `hotstart` function calls the input argument `nWSR` is mandatory. Additionally, it is possible to specify a maximum amount of CPU time to be spent on the respective QP solution. For doing so, a non-null pointer to a `double` containing the maximum allowed CPU time in seconds needs to be specified. If both, a maximum number of working set recalculations `nWSR` and a maximum allowed CPU time `cputime` is given, the solution procedure stops as soon as *one of these limits* is reached, whatever may occur first.

The CPU time limitation is based on a *heuristic* that estimates the required CPU time for the next working set change; if there is not enough time left, the solution procedure stops. This heuristic is based on the CPU time measurements of the previous working set changes, thus the actual total CPU time might be slightly higher that the allowed one due to time measurement inaccuracies. However, it is guaranteed that *at most one* working set change too much is performed.

Note that the CPU timit limit only can take effect if a system clock is available via the global `getCPUtime` function (implemented within the file `src/Utils.cpp`).

5.7 Further Useful Functionality

Reading Data From Files

Both the `init` and the `hotstart` functions are overloaded with variants that are able to read the required data directly from a plain ASCII file, e.g.:

- ```
  returnValue init( const char* const H_file,
                    const char* const g_file,
                    const char* const A_file,
                    const char* const lb_file,
                    const char* const ub_file,
                    const char* const lbA_file,
                    const char* const ubA_file,
                    int&              nWSR,
                    double* const     cputime
                    );
  ```

- ```
  returnValue hotstart( const char* const g_file,
                        const char* const lb_file,
                        const char* const ub_file,
                        const char* const lbA_file,
                        const char* const ubA_file,
                        int&              nWSR,
                        double* const     cputime
                        );
  ```

Instead of a `double` array, they expect a string with the name of the ASCII file containing the respective data. Data files must be stored row-wise; all entries within one row should be space- or tabulator-separated.

These variants also exists for the case when an initial guess for the active set is provided (as described in Section 5.5).

Output Settings

You can adjust the text output of qpOASES using the following functions:

- `PrintLevel getPrintLevel( ) const`,
- `void setPrintLevel( PrintLevel _printlevel )`.

The function `getPrintLevel` returns one of the following print levels:

- `PL_NONE`: no output at all,
- `PL_LOW`: print error messages only,
- `PL_MEDIUM`: print error messages, warnings, some info messages as well as a concise iteration summary (default value),
- `PL_HIGH`: print all messages that occur while iterating.

By means of the function `setPrintLevel` you can specify one of the above-mentioned print levels whenever desired.

Resetting a `QProblem` Object

Sometimes it can be useful to reset an existing `QProblem` object. This is particularly helpful if you want to restart while solving a QP sequence (e.g. after an internal error has occured) without creating a new object. This feature is provided by the following function:

```
returnValue reset( );
```

It resets all internal data structures and matrix factorisations and thus leaves the `QProblem` object in exactly the same state as it would be after a constructor call. Therefore, you need to call an `init` function for solving the first QP after an execution of `reset`.

Printing QP Properties and Options

At any time you might print a concise list of properties of the QP object by calling:

```
returnValue printProperties( );
```

Besides other information, it displays number and type of bounds and constraints, respectively, the type of the Hessian matrix as well as the status of the QP object.

Moreover, a list of all options and their current values (see Section 5.2) can be printed as follows:

```
returnValue printOptions( );
```

5.8 Add-Ons for qpOASES

When compiling the source code of qpOASES, a second library libqpOASESextras.a is created. Its functionality comprises all the functionality of the standard libqpOASES.a and additionally provides several add-ons which are described in the following subsections. Header and implementation files of these add-ons are located within a sub-folder EXTRAS of include and src, respectively.

5.8.1 Solution Analysis

For a posteriori analysis of a QP solution the SolutionAnalysis class is provided as an add-on to qpOASES. Currently it implements the following two functions:

- Determination of the maximum violation of the KKT optimality conditions:

```
returnValue getMaxKKTviolation( QProblem* qp,
                                double& maxKKTviolation
                                ) const;
```

  This function takes a pointer to a QProblem object which is assumed to have readily solved an (intermediate) QP and writes the maximum violation of the KKT optimality conditions into the argument maxKKTviolation. If the QProblem object has not solved the current QP, the status code RET_UNABLE_TO_ANALYSE_QPROBLEM is returned.

- Computation of the variance-covariance matrix of the QP output for uncertain inputs:

```
returnValue getVarianceCovariance( QProblem* qp,
                                   double* g_b_bA_VAR,
                                   double* Primal_Dual_VAR
                                   ) const;
```

  It also takes a QProblem object which is assumed to have readily solved an (intermediate) QP as well as the variance-covariance of the gradient, the bounds and the constraints' bounds, respectively (matrix dimension: $2\mathtt{nV}+\mathtt{nC} * 2\mathtt{nV}+\mathtt{nC}$). The variance-covariance matrix of the primal and dual variables is written into the argument Primal_Dual_VAR (matrix dimension: $2\mathtt{nV}+\mathtt{nC} * 2\mathtt{nV}+\mathtt{nC}$), which needs to be allocated by the user.

For using the SolutionAnalysis class you need to include its header SolutionAnalysis.hpp into your source file, a complete example can be found in the file <install-dir>/examples/example2.cpp.

5.8.2 Solving Test Problems from the Online QP Benchmark Collection

A second qpOASES add-on is intended to facilitate the solution of test problems from the Online QP Benchmark Collection [1]. Data for a whole QP sequence with constant matrices along with its optimal primal/dual solution vectors and the optimal objective function value is stored in plain ASCII files. For conveniently reading these files, three functions are provided (see <install-dir>/include/EXTRAS/OQPinterface.hpp for a detailed documentation):

- readOQPdimensions for reading the dimensions of the QP sequence,
- readOQPdata for reading data and solution information of the QP sequence,
- solveOQPbenchmark for solving a given benchmark QP sequence.

Moreover, the following function summarises the functionality of the three above-mentioned ones:

```
returnValue runOQPbenchmark( const char* path,
                             BooleanType isSparse,
                             const Options& options,
                             int& nWSR,
                             double& maxCPUtime,
                             double& maxStationarity,
                             double& maxFeasibility,
                             double& maxComplementarity
                             );
```

It takes the path to the directory where the benchmark problem is stored as first argument. Second, the user can specify whether the QP matrices shall be converted to the sparse matrix format before solution. Moreover, user-defined QP solver options and the maximum number of working set recalculations per QP are passed as input arguments. On output nWSR contains the maximum number of working set recalculations that have been actually performed, maxCPUtime contains the maximum CPU time that have been required for solving each of the QPs. maxStationarity, maxFeasibility, maxComplementarity contain the maximum violations of the optimality conditions with respect to stationarity, feasibility and complementary of the obtained QP solutions, respectively.

For using this add-on you need to include the header file OQPinterface.hpp into your source code, a complete example can be found in the file <install-dir>/examples/example3.cpp. In order to run this example, you need to download example no. 01 from the Online QP Benchmark Collection website [1] first and extract its archive into the sub-folder <install-dir>/examples/chain80w/.

Chapter 6

Interfaces for Third-Party Software

If you want to use qpOASES via one of the following third-party interfaces, make sure that you have performed the installation steps 1 through 3 from Chapter 2. Afterwards, proceed with the installation of the desired interface as described in this chapter.

6.1 Interface for Matlab

Installation

It is possible to use qpOASES directly within the MATLAB environment. This is facilitated by compiling it into a so-called MEX function, which can be done as follows:

1. Start MATLAB and run `mex -setup` for choosing a C++ compiler (e.g. `gcc`).

2. Execute the following commands:

   ```
   cd <install-dir>/interfaces/matlab
   make
   ```

   The latter command runs the MATLAB script `make.m` which does the compilation. Executables `qpOASES.<ext>`, `qpOASES_sequence.<ext>`, `qpOASES_sequenceSB.<ext>` and `qpOASES_sequenceVM.<ext>` should be created, where `<ext>` (e.g. `mexglx`) depends on your operating system.

Remarks:

- The compilation was tested under LINUX using MATLAB 7.3 and higher together with the `gcc` compiler. Modifications of the `make.m` script might be necessary depending on your operating system, your MATLAB version and your compiler. For compiling the MATLAB interface for WINDOWS operating systems, the MICROSOFT® Visual C++ 2008 Express Edition has proven to work.

- If compilation fails due to the fact that the `snprintf()` function is not supported, you might uncomment line 41 within `<install-dir>/include/Types.hpp` and try to compile again.

Interface for Solving a Single QP

After a successful installation, you can call qpOASES as conventional QP solver from the MATLAB environment (using a cold start every time):

```
[x,fval,exitflag,iter,lambda] = qpOASES( H,g,A,lb,ub,lbA,ubA,{x0,{options}} )
```

This command combines the creation of a `QProblem` object and a calls to the function `init` (see Chapter 3): the *input arguments*[1] specify the Hessian matrix, the gradient vector, the constraint matrix, the lower and upper bound vectors, the lower and upper constraints' vectors, respectively. Again, the Hessian has to be symmetric and positive definite and all vectors must be stored as column vectors. Optionally, an initial guess for the primal solution (cf. Section 5.5) can be specified and a set of options can be passed. It is possible to leave one or more of the input arguments `lb`, `ub`, `lbA`, `ubA` empty if your QP formulation does not comprise the corresponding limits.
If no initial guess is given, the usual homotopy starting at the origin is performed. Options can be generated using the `qpOASES_options` command. Called without arguments, it generates a struct containing all options as described in Section 5.2. For changing these values, two equivalent possibilities exist (see the MATLAB help for more details):

```
// change default values when creating options struct...
myOptions = qpOASES_options( 'printLevel',2, 'enableFlippingBounds',0 )

// or change them later
myOptions = qpOASES_options
myOptions.printLevel = 2
myOptions.enableFlippingBounds = 0
```

In addition to the options described in Section 5.2, the MATLAB options struct also contains the entry `maxIter` for specifying the maximum number of iterations (corresponds to `nWSR` in the C++ version). If it is not set by the user, the default value $5 * (\mathtt{nV} + \mathtt{nC})$ is chosen.

The *output arguments* contain the optimal primal solution vector, the optimal objective function value, a status flag, the number of iterations actually performed, and the optimal dual solution vector, respectively. The status flag can take one of the following values:

- `0`: QP was solved,
- `1`: QP could not be solved within the given number of working set recalculations,
- `-1`: QP could not be solved due to an internal error,
- `-2`: QP is infeasibile and thus could not be solved,
- `-3`: QP is unbounded and thus could not be solved.

If you do not need all output information, you can leave all but the first one away, e.g.

```
[x,fval] = qpOASES( H,g,A,lb,ub,[],ubA )
```

[1] matrices can be passed either in dense or sparse matrix format

Remark: The function qpOASES also allows you to solve a pre-computed sequence of QPs with fixed matrices: you just have to pass a whole sequence of input vectors. Each vector must be stored column-wise in a matrix, i.e. the ith QP is given by the ith columns of the QP "vectors" g, lb, ub, lbA, ubA, and all these five matrices must have the same number of columns. As both the Hessian and the constraint matrix remain constant, they are passed as in the case of a single QP. If a whole sequence of QPs is to be solved, also the outputs are given column-wise, i.e. x is a matrix with optimal primal solution vectors stored column-wise inside, fval is a row vector, and so on.

The interface allows you to directly use the QProblemB class for simply bounded QPs (cf. Section 4.3) by simply leaving the arguments A, lbA, ubA away:

```
[x,fval,exitflag,iter,lambda] = qpOASES( H,g,lb,ub,{x0,{options}} )
```

Again, a default value for the number of working set recalculations is used (here $5 * \mathtt{nV}$) if maxIter is not specified within options. Also here you can leave lb or ub empty if they do not occur within your QP formulation.

Interface for Solving a QP Sequence

As the online active set strategy is intended to solve a whole sequence of parameterised QPs, there exist a special MATLAB function for hotstarting each QP from the solution of the previous one:

```
[x,fval,exitflag,iter,lambda] = qpOASES_sequence( 'i',H,g,A,lb,ub,lbA,ubA,{x0,{options}} )
[x,fval,exitflag,iter,lambda] = qpOASES_sequence( 'h',g,lb,ub,lbA,ubA,{options} )
                                qpOASES_sequence( 'c' )
```

As in the C++ implementation (cf. Chapter 3), the first QP of the sequence is solved together with the initialisation all internal data structures. For this purpose, the function qpOASES_sequence (called with first input argument 'i') takes all QP data and optionally the maximum number of working set recalcutations for solving the initial QP and an initial primal solution guess as further input arguments. It provides the usual output information (see above) and you can leave all but the first output argument away, again.
Afterwards, each subsequent QP can be solved by performing a so-called "hot start" using the function qpOASES_sequence, again (this time called with first input argument 'h'). It takes the QP vectors of the new QP as well as the maximum number of working set recalcutations as further input arguments, and provides the usual output information.
Having solved the last QP of the sequence, you are encouraged to free the internal memory by calling qpOASES_sequence('c').

For solving QPs of special types as described in Chapter 4, special variants of the above function are provided: first, you can run the commands

```
[x,fval,exitflag,iter,lambda] = qpOASES_sequenceSB( 'i',H,g,lb,ub,{x0,{options}} )
[x,fval,exitflag,iter,lambda] = qpOASES_sequenceSB( 'h',g,lb,ub,{options} )
                                qpOASES_sequenceSB( 'c' )
```

for solving *s*imply *b*ounded QPs (input arguments correspoding to constraints are simply left away); second, call

```
[x,fval,exitflag,iter,lambda] = qpOASES_sequenceVM( 'i',H,g,A,lb,ub,lbA,ubA,{x0,{options}} )
[x,fval,exitflag,iter,lambda] = qpOASES_sequenceVM( 'h',H,g,A,lb,ub,lbA,ubA,{options} )
                                qpOASES_sequenceVM( 'c' )
```

for solving QPs with *v*arying *m*atrices, where qpOASES_sequenceVM also takes the new matrices of the next QP of the sequence. Again, the internal memory is freed by calling qpOASES_sequenceSB('c') and qpOASES_sequenceVM('c'), respectively. This memory is kept independently for all three QP types.

Examples

The files example1.mat, example1a.mat and example1b.mat contain, respectively, very basic examples for solving a sequence comprising two QPs with fixed matrices, varying matrices, and with simple bounds only. For solving the first one do the following:

1. Start MATLAB and execute the following commands:

   ```
   cd <install-dir>/interfaces/matlab
   load example1.mat
   ```

2. Solve the first QP by typing

   ```
   options = qpOASES_options( 'maxIter',10 );
   [x,fval,exitflag,iter,lambda] =
         qpOASES_sequence( 'i',H,g,A,lb,ub,lbA,ubA,[],options )
   ```

3. Solve the second QP by typing

   ```
   [x,fval,exitflag,iter,lambda] =
         qpOASES_sequence( 'h',g_new,lb_new,ub_new,lbA_new,ubA_new,options )
   ```

4. Free the internal memory by calling

   ```
   qpOASES_sequence( 'c' )
   ```

6.2 Interface for Simulink

Installation

You can use qpOASES directly within the SIMULINK environment, too. This requires to compile it into a so-called S function, which can be done as follows:

1. Start MATLAB and run mex -setup for choosing a C++ compiler (e.g. gcc).

2. Execute the following commands:

   ```
   cd <install-dir>/interfaces/simulink
   make
   ```

   The latter command runs the MATLAB script make.m which does the compilation. Three executables called qpOASES_QProblemB.<ext>, qpOASES_QProblem.<ext> and qpOASES_SQProblem.<ext> should be created, where <ext> (e.g. mexglx) depends on your operating system.

Remarks:

- The compilation was tested under LINUX using MATLAB 7.3 and higher together with the gcc compiler. Modifications of the `make.m` script might be necessary depending on your operating system, your MATLAB version and your compiler. For compiling the MATLAB interface for WINDOWS operating systems, the MICROSOFT® Visual C++ 2008 Express Edition has proven to work.
- If compilation fails due to the fact that the `snprintf()` function is not supported, you might uncomment line 41 within `<install-dir>/include/Types.hpp` and try to compile again.

Interface

There exist three different S function interfaces corresponding to the three different types of QP sequences to be solved (see also Chapter 4):

1. `qpOASES_QProblemB.<ext>` for solving simply bounded QPs,
2. `qpOASES_QProblem.<ext>` for solving QPs with fixed matrices,
3. `qpOASES_SQProblem.<ext>` for solving QPs with varying matrices.

For each of these interfaces a simple example is provided within the folder `<install-dir>/interfaces/simulink`. We only give details for the one for QPs with fixed matrices, as the other ones work analoguously.

In order to run the example, start MATLAB and execute the corresponding script file as follows:

```
cd <install-dir>/interfaces/simulink
load_example_QProblem
```

The sample QP data is loaded into the workspace and the file `qpOASES_QProblem.mdl` (depicted in Figure 6.1) is opened.

The `qpOASES` S function has *seven inputs*:

- the (fixed) QP matrices `H` and `A` as well as
- the QP vectors `g`, `lb`, `ub`, `lbA`, `ubA`, which can be updated at each sampling instant.

The dimensions of the inputs are detected automatically, but they have to be consistent (e.g. the dimension of `H` needs to be the squared size of `g`).

Moreover, you have to *define three additional values* near top of the file `qpOASES_QProblem.cpp` *before compilation of the S function*:

- `#define SAMPLINGTIME <value>`: the sample time of the SIMULINK block,
- `#define NCONTROLINPUTS <value>`: the number of control inputs of your system (the leading `NCONTROLINPUTS` components of the optimal primal solution vector are returned as optimal output by the S function),

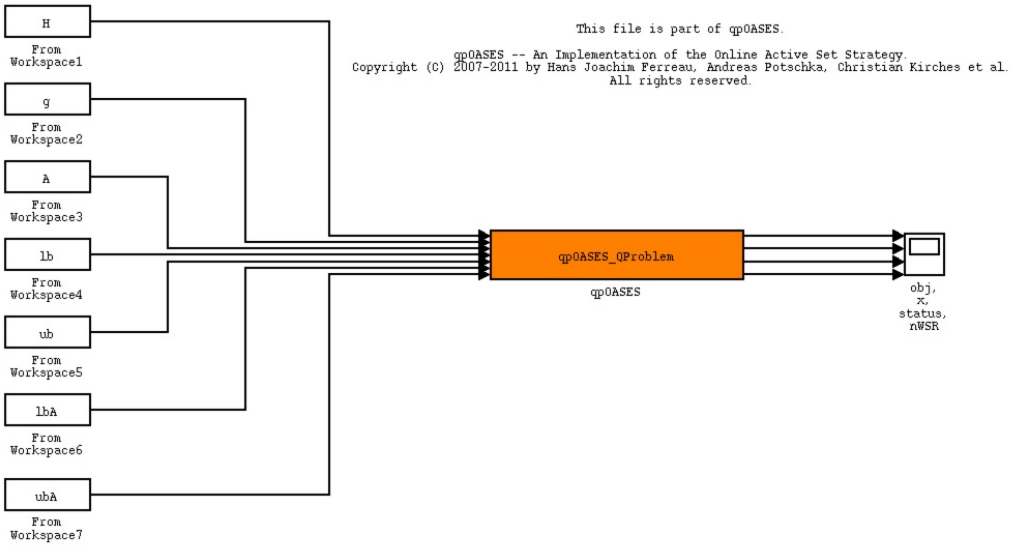

Figure 6.1: qpOASES working as SIMULINK S function.

- `#define NWSR <value>`: the maximum number of working set recalculations to be performed per QP.

For running the example you can use the specified default values; but do not forget to adjust them to the requirements of your own problem.

At each sampling instant the qpOASES S function provides the following *four outputs*:

- `obj`: the optimal objective function value;
- `x`: the leading `NCONTROLINPUTS` components of optimal primal solution vector;
- `status`: a status flag which can take one of the following values
  - 0: QP was solved,
  - 1: QP could not be solved within the given number of working set recalculations,
  - -1: QP could not be solved due to an internal error,
  - -2: QP is infeasibile and thus could not be solved,
  - -3: QP is unbounded and thus could not be solved;
- `nWSR`: the number of working set recalculations actually performed.

An Example

Having executed the script `load_example_QProblem` as described above, you can simply start the SIMULINK simulation given by the file `example_QProblem.mdl`. The simulation runs for 0.5 s with a sample time of 0.1 s. At the first two sampling instants the QPs as specified in the file `example1.mat` of the MATLAB interface are solved; at the remaining

sampling instants the last QP is solved repeatedly (requiring zero iterations as the hotstart feature of the online active set strategy is used).

6.3 Interface for Octave

An interface for using qpOASES within OCTAVE will be made available soon.

6.4 Interface for scilab

Installation

For using qpOASES within SCILAB, you have to perform the following steps:

1. Compile the SCILAB interface by executing the following commands:

   ```
   cd <install-dir>/interfaces/scilab
   make
   ```

2. Start SCILAB and link the interface to the SCILAB environment:

   ```
   exec qpOASESinterface.sce;
   ```

Interface for Solving a Single QP

If you simply want to use qpOASES as conventional QP solver (using a cold start every time), you can call it as follows:

```
[x,fval,exitflag,iter,lambda] = qpOASES( H,g,A,lb,ub,lbA,ubA,nWSR )
```

The *input arguments* specify the Hessian matrix, the gradient vector, the constraint matrix, the lower and upper bound vectors, the lower and upper constraints' vectors, and the maximum number of working set recalculations, respectively. As usual, the Hessian must be symmetric and positive definite and all vectors must be stored as column vectors.
The *output arguments* contain the optimal primal solution vector, the optimal objective function value, a status flag, the number of iterations actually performed, and the optimal dual solution vector, respectively. The status flag can take one of the following values:

- 0: QP was solved,
- 1: QP could not be solved within the given number of working set recalculations,
- -1: QP could not be solved due to an internal error,
- -2: QP is infeasibile and thus could not be solved,
- -3: QP is unbounded and thus could not be solved.

If you do not need all output information, you can leave all but the first one away.

Remark: A special variant for simply bounded QPs is not yet interfaced.

Interface for Solving a QP Sequence

As the online active set strategy is intended to solve a whole sequence of parameterised QPs, there exist special routines for doing so:

```
[x,fval,exitflag,iter,lambda] = qpOASES_init( H,g,A,lb,ub,lbA,ubA,nWSR )
[x,fval,exitflag,iter,lambda] = qpOASES_hotstart( g,lb,ub,lbA,ubA,nWSR )
                                qpOASES_cleanup
```

As in the C++ implementation (cf. Chapter 3), the first QP of the sequence is solved together with the initialisation all internal data structures. For this purpose, the function `qpOASES_init` takes all QP data and the maximum number of working set recalcutations for solving the initial QP as input arguments, and provides the usual output information (see above).
Afterwards, each subsequent QP is can be solved by performing a so-called "hot start" using the function `qpOASES_hotstart`. It takes the QP vectors of the new QP as well as the maximum number of working set recalcutations as input arguments, and provides the usual output information, again.
Having solved the last QP of the sequence, you are encouraged to free the internal memory by calling `qpOASES_cleanup`.

For solving QPs of special types as described in Chapter 4, special variants of the above functions are provided. First, the functions

```
[x,fval,exitflag,iter,lambda] = qpOASES_initSB( H,g,lb,ub,nWSR )
[x,fval,exitflag,iter,lambda] = qpOASES_hotstartSB( g,lb,ub,nWSR )
                                qpOASES_cleanupSB
```

for *s*imply *b*ounded QPs (input arguments correspoding to constraints are simply left away). Second, the functions

```
[x,fval,exitflag,iter,lambda] = qpOASES_initVM( H,g,A,lb,ub,lbA,ubA,nWSR )
[x,fval,exitflag,iter,lambda] = qpOASES_hotstartVM( H,g,A,lb,ub,lbA,ubA,nWSR )
                                qpOASES_cleanupVM
```

for QPs with *v*arying *m*atrices, where `qpOASES_hotstartVM` also takes the new matrices of the next QP of the sequence.

Again, the internal memory is freed by calling `qpOASES_cleanupSB` and `qpOASES_cleanupVM`, respectively. This memory is kept independently for all three QP types.

Examples

The files `example1.dat`, `example1a.dat` and `example1b.dat` contain, respectively, very basic examples for solving a sequence comprising two QPs with fixed matrices, varying matrices, and with simple bounds only. For solving the first one do the following:

1. Start SCILAB and execute the following commands:

   ```
   cd <install-dir>/interfaces/scilab
   load('example1.dat')
   ```

2. Solve the first QP by typing

   ```
   [x,fval,exitflag,iter,lambda] =
            qpOASES_init( H,g,A,lb,ub,lbA,ubA,10 )
   ```

3. Solve the second QP by typing

   ```
   [x,fval,exitflag,iter,lambda] =
            qpOASES_hotstart( g_new,lb_new,ub_new,lbA_new,ubA_new,10 )
   ```

4. Free the internal memory by calling qpOASES_cleanup.

6.5 Running qpOASES on dSPACE

qpOASES can be easily run on a dSPACE board via its Simulink interface, provided that a C++ compiler is available. This has been tested for dSPACE boards version 5.3 or higher together with the dSPACE C++ Integration Kit 1.0.2 or higher. The following additional notes hopefully facilitate the setup:

1. Setup your dSPACE system
2. Install the dSPACE C++ Integration Kit
3. Install qpOASES (its Simulink interface, to be more precisely)
4. Compile qpOASES with compiler flag __DSPACE__. This can be done, e.g., by uncommenting line 45 within <install-dir>/include/Types.hpp.
5. Setup your Simulink project
6. Open MK(make) file of your project (eventually you have to compile it once before) and add the following lines at the head of this file:

   ```
   # enable c++ support
   USER_BUILD_CPP_APPL = ON
   ```

7. Also complete the following lines:

   ```
   USER_SRCS = qpOASES_SQProblem.cpp qpOASES_QProblem.cpp
   qpOASES_QProblemB.cpp SQProblem.cpp QProblem.cpp QProblemB.cpp
   Bounds.cpp Constraints.cpp SubjectTo.cpp Indexlist.cpp
   Flipper.cpp Utils.cpp Options.cpp Matrices.cpp
   BLASReplacement.cpp LAPACKReplacement.cpp MessageHandling.cpp
   ```
   (i.e. all source files of qpOASES and its Simulink interface)

   ```
   USER_SRCS_DIR = ./src
   ```
   (i.e. directory of qpOASES source files)

   ```
   USER_INCLUDES_PATH = ./include ./src
   ```
   (i.e. directories of qpOASES header and source files)

8. Compile your project
9. Run the compiled project on dSPACE

6.6 Using qpOASES within the ACADO Toolkit

ACADO Toolkit is a software framework for automatic control and dynamic optimisation available at

http://www.acadotoolkit.org.

It is an open-source (LGPL) environment for setting up a great variety of dynamic optimization problems for use in control, in particular (nonlinear) model predictive control. ACADO Toolkit uses qpOASES as default QP solver, for linear MPC as well as for the QP sequences resulting from SQP-type methods.

6.7 Using qpOASES within MUSCOD-II

MUSCOD-II is a proprietary software package for numerical solution of optimal control problems involving differential-algebraic equations, developed by the members of the "Simulation and Optimization Group" of the Interdisciplinary Center for Scientific Computing (IWR) at University of Heidelberg. The current version of MUSCOD-II also contains an interface for using qpOASES as underlying QP solver.

Chapter 7

Developer Information and Compiling Options

This chapter provides a very brief introduction to the qpOASES software design. If you are interested in using qpOASES within your own software project or in developing extensions for it yourself, we recommend to consult its DOXYGEN documentation (cf. installation step six described in Chapter 2) for detailed information. Moreover, you are encouraged to pose questions or remarks to `support@qpOASES.org`.

7.1 Class Hierarchy

So far, we mainly mentioned four different classes: `QProblem`, `QProblemB` `SQProblem` and `Options`. These are the only classes which provide user interfaces for accessing qpOASES's functionality. However, they are not the only classes of the qpOASES software package but are embedded in a more complex hierarchy.

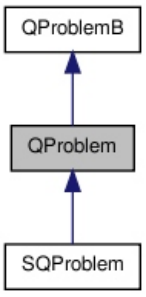

Figure 7.1: `QProblem` class hierarchy (illustrated with DOXYGEN [7]).

The class `QProblemB` is at the bottom of the hierarchy (see Figure 7.1) and provides all functionality necessary for solving a simply bounded quadratic program (cf. Section 4.3). The `QProblem` class is derived from it and implements all necessary additional functionality for solving a QPs comprising general constraints. The class `SQProblem`, in turn, inherits all features of the `QProblem` class and provides further functionality for handling QPs with varying matrices (cf. Section 4.2).

All the three classes QProblemB, QProblem and SQProblem make use of further auxiliary classes: First, they have a member of type Options to store user-defined QP solver options. Second, they hold members of type Bounds or Constraints (which are derived from a common type SubjectTo) in order to store bounds or constraints of a QP. Both the Bounds and the Constraints class manages lists (of type Indexlist) of free and fixed variables and active and inactive constraints, respectively. Third, they hold an instance of the Flipper class for storing a temporary copy of the matrix factorisations whenever necessary. Finally, they hold pointers to the matrices of the current QP and to a user-defined ConstraintProduct definition (see Section 5.4). QP matrices are stored within one of the classes depicted in Figure 7.2 depending on their structure.

All the above mentioned classes use a class called MessageHandling for providing errors messages, warnings or other information to the user and for handling return values of their member functions in a unified framework. This class makes use of the enumeration returnValue, which gathers all possible return values of all qpOASES functions. The current implementation uses a single *global* instance of the MessageHandling class; the global function

```
MessageHandling* getGlobalMessageHandler( );
```

returns a pointer to it.

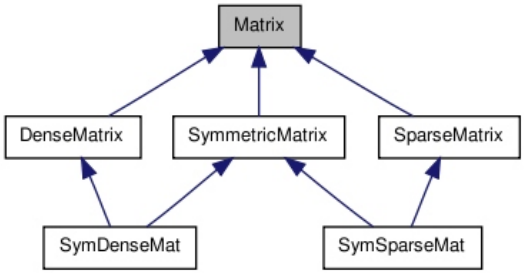

Figure 7.2: qpOASES matrix class hierarchy (illustrated with DOXYGEN [7]).

7.2 Global Constants

Some useful global constants are defined in file <install-dir>/include/Constants.hpp. Their default values seem to work reasonably, but you might change them if necessary:

- EPS: numerical value of machine precision,
- ZERO: numerical value of zero (for situations in which it would be unreasonable to compare with 0.0),
- INFTY: numerical value of infinity (e.g. for non-existing bounds),

7.3 Compiler Flags

When compiling qpOASES, you can define the following compiler flags:

- LINUX : activates LINUX-specific functionality (e.g. time measurement),
- WIN32 : activates WINDOWS-specific functionality (e.g. time measurement),
- __MATLAB__ : activates MATLAB-specific functionality (in particular, the use of mexPrintf instead of printf),
- __cplusplus : necessary for building C++ S functions for SIMULINK,
- __DSPACE__ : define this compiler flag in order to disable the qpOASES namespace (and switching off all text messages) for ensuring backward compatibility with dSPACE compilers,
- __XPCTARGET__ : define this compiler flag in order to disable all text messages for ensuring compatibility for xPC TARGET compilers,
- __DEBUG__ : activates more detailed output messages during QP solution,
- __DEBUG_ITER__ : activates a detailed iteration output during QP solution,
- __SUPPRESSANYOUTPUT__ : suppresses any console output during QP solution,
- __NO_COPYRIGHT__ : suppresses copyright notice at beginning of QP solution,
- __ALWAYS_INITIALISE_WITH_ALL_EQUALITIES__ : forces to always include all implicitly fixed bounds and all equality constraints into the initial working set when setting up an auxiliary QP,
- __MANY_CONSTRAINTS__ : enables a usually faster way for determining the current step length for QPs comprising many constraints (see Section 5.4),
- __USE_THREE_MULTS_GIVENS__ : switches to a different way of calculating Givens rotations that requires only three multiplications,
- __USE_SINGLE_PRECISION__ : switches to single precision arithmetic.

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Appendix A

qpOASES Software Licence

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```
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```

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  If distribution of object code is made by offering access to copy
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